KUANT Guides

Guide No.
KUANT 024.1

Tracing Rules in Structural Equation Modeling:

Ge, F., & Childress, S., (2010)

Preliminaries:

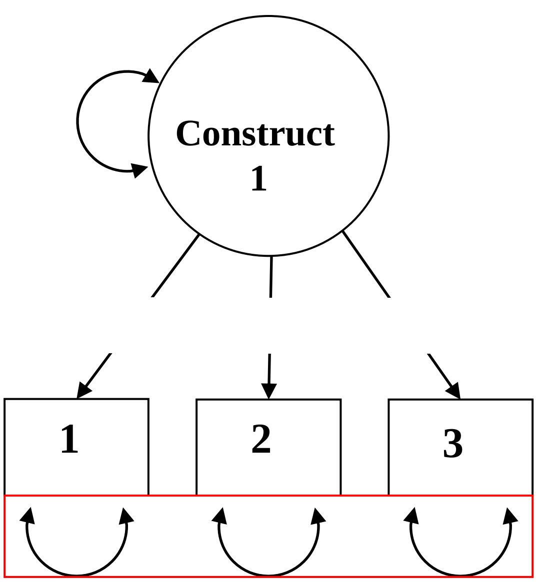
This handout offers an outline of how to calculate the expected variances, covariances and means in a two-factor CFA model. All examples are presented with a path diagram illustration and mathematical decomposition. Since different drawing and labeling conventions of path diagrams are utilized in SEM modeling, this handout offers examples in two mostly used drawing conventions for SEM: 1) A simpler short-hand way; 2) A technically accurate way.

Tracing Rules for Variances/Covariances*

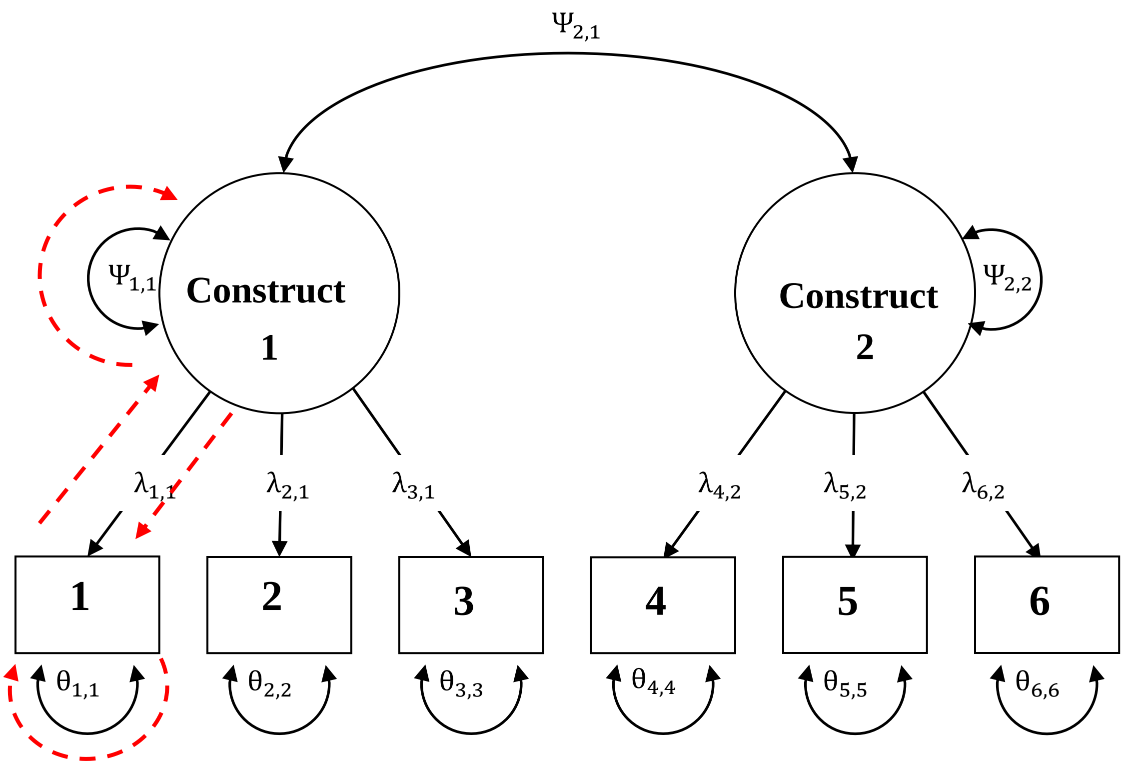
- Traces all paths between indicators multiplying as you go.
- "Arrows" are unidirectional (start at the point): no going forward and then backward.
- No loops are allowed:
  - When tracing from one variable to another, you cannot pass through the same variable twice.
  - Only ONE curved arrow is allowed in one tracing.
  - Add all paths together.

*Details are available at here (need a KU ID to get access).

Convention 1:



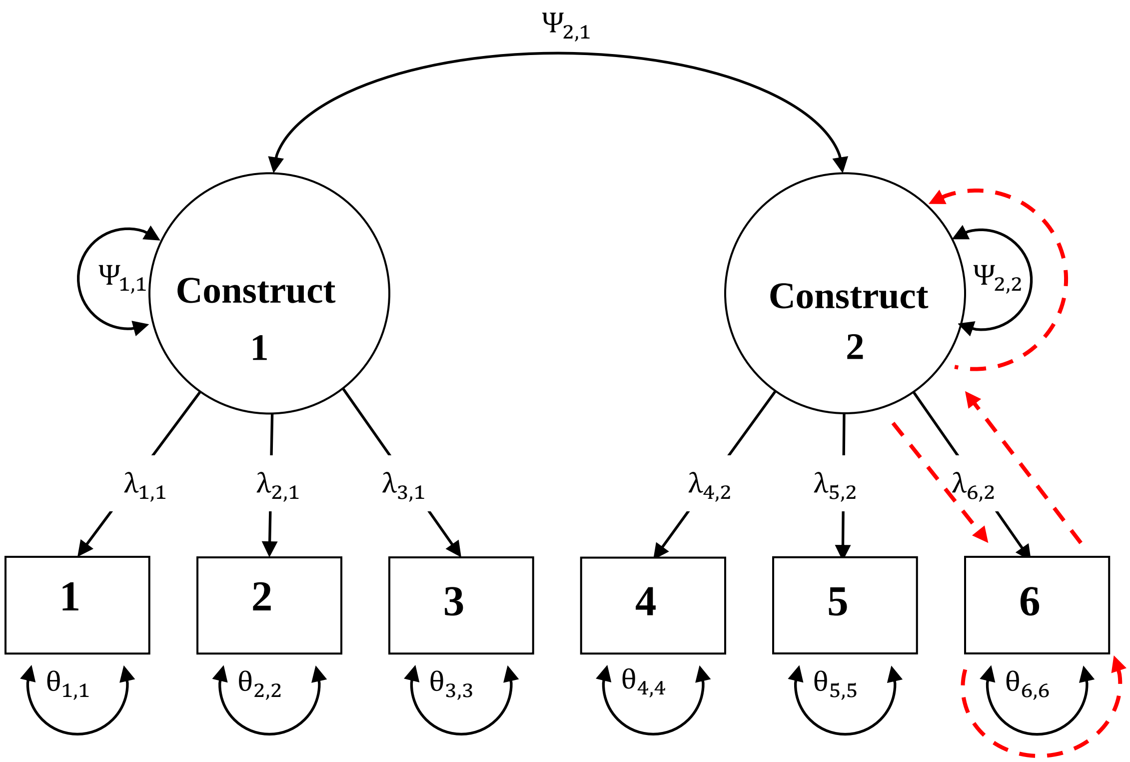
A preferred short-hand way: the residual-variance information is represented as "unexplained" variance in the indicator.



Variance of "indicator 1" = $\lambda_{1,1}\Psi_{1,1}\lambda_{1,1} + \theta_{1,1}$

Σ = Model Implied Correlation/Covariance Matrix

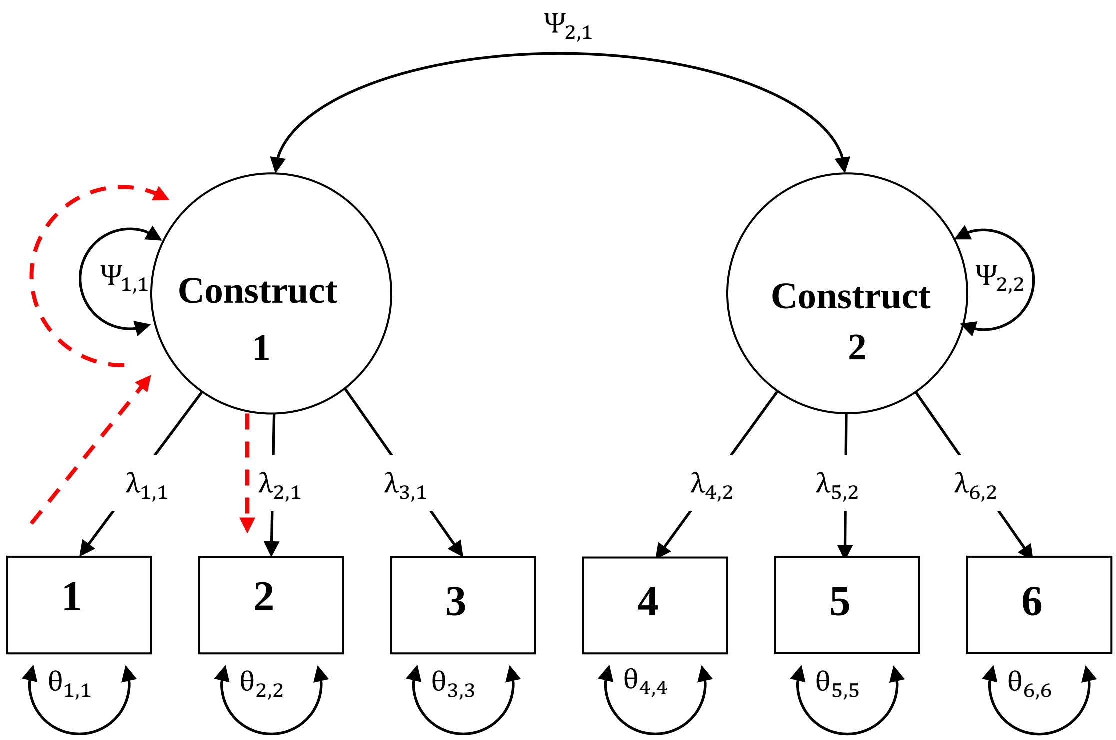
	Indicator1	Indicator2	Indicator3	Indicator4	Indicator5	Indicator6
C1,1	$\lambda_{1,1}\Psi_{1,1}\lambda_{1,1}+\theta_{1,1}$					
C1,2	$\lambda_{1,1}\Psi_{1,1}\lambda_{2,1}$	$\lambda_{2,1}\Psi_{1,1}\lambda_{2,1}+\theta_{2,2}$				
C1,3	$\lambda_{1,1}\Psi_{1,1}\lambda_{3,1}$	$\lambda_{2,1}\Psi_{1,1}\lambda_{3,1}$	$\lambda_{3,1}\Psi_{1,1}\lambda_{3,1}+\theta_{3,3}$			
C2,1	$\lambda_{1,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{2,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{3,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{4,2}\Psi_{2,2}\lambda_{4,2}+\theta_{4,4}$		
C2,2	$\lambda_{1,1}\Psi_{2,1}\lambda_{5,2}$	$\lambda_{2,1}\Psi_{2,1}\lambda_{5,2}$	$\lambda_{3,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{4,2}\Psi_{2,2}\lambda_{5,2}$	$\lambda_{5,2}\Psi_{2,2}\lambda_{5,2}+\theta_{5,5}$	
C2,3	$\lambda_{1,1}\Psi_{2,1}\lambda_{6,2}$	$\lambda_{2,1}\Psi_{2,1}\lambda_{6,2}$	$\lambda_{3,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{4,2}\Psi_{2,2}\lambda_{6,2}$	$\lambda_{5,2}\Psi_{2,2}\lambda_{6,2}$	$\lambda_{6,2}\Psi_{2,2}\lambda_{6,2}+\theta_{6,6}$



Variance of "indicator 6" = $\lambda_{6,2}\Psi_{2,2}\lambda_{6,2} + \theta_{6,6}$

Σ = Model Implied Correlation/Covariance Matrix

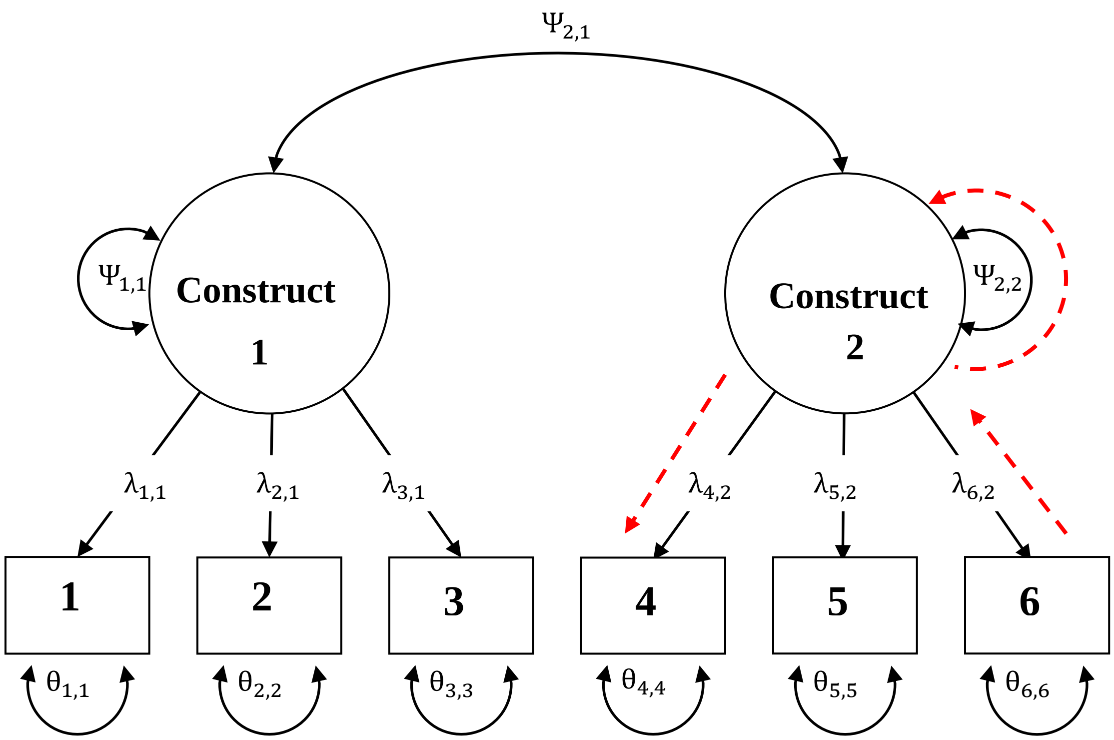
	Indicator1	Indicator2	Indicator3	Indicator4	Indicator5	Indicator6
C1,1	$\lambda_{1,1}\Psi_{1,1}\lambda_{1,1}+\theta_{1,1}$					
C1,2	$\lambda_{1,1}\Psi_{1,1}\lambda_{2,1}$	$\lambda_{2,1}\Psi_{1,1}\lambda_{2,1}+\theta_{2,2}$				
C1,3	$\lambda_{1,1}\Psi_{1,1}\lambda_{3,1}$	$\lambda_{2,1}\Psi_{1,1}\lambda_{3,1}$	$\lambda_{3,1}\Psi_{1,1}\lambda_{3,1}+\theta_{3,3}$			
C2,1	$\lambda_{1,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{2,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{3,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{4,2}\Psi_{2,2}\lambda_{4,2}+\theta_{4,4}$		
C2,2	$\lambda_{1,1}\Psi_{2,1}\lambda_{5,2}$	$\lambda_{2,1}\Psi_{2,1}\lambda_{5,2}$	$\lambda_{3,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{4,2}\Psi_{2,2}\lambda_{5,2}$	$\lambda_{5,2}\Psi_{2,2}\lambda_{5,2}+\theta_{5,5}$	
C2,3	$\lambda_{1,1}\Psi_{2,1}\lambda_{6,2}$	$\lambda_{2,1}\Psi_{2,1}\lambda_{6,2}$	$\lambda_{3,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{4,2}\Psi_{2,2}\lambda_{6,2}$	$\lambda_{5,2}\Psi_{2,2}\lambda_{6,2}$	$\lambda_{6,2}\Psi_{2,2}\lambda_{6,2}+\theta_{6,6}$



Covariance between “indicator 1” and “indicator 2” = $\lambda_{1,1}\Psi_{1,1}\lambda_{2,1}$

Σ = Model Implied Correlation/Covariance Matrix

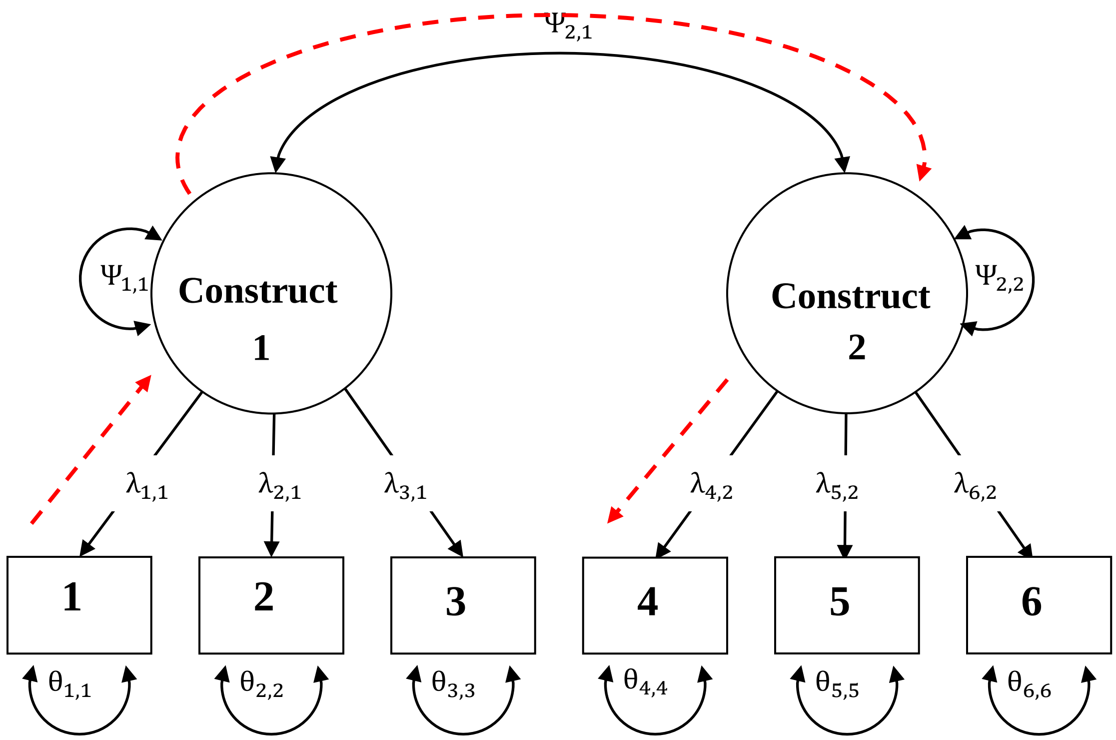
	Indicator1	Indicator2	Indicator3	Indicator4	Indicator5	Indicator6
C1,1	$\lambda_{1,1}\Psi_{1,1}\lambda_{1,1}+\theta_{1,1}$					
C1,2	$\lambda_{1,1}\Psi_{1,1}\lambda_{2,1}$	$\lambda_{2,1}\Psi_{1,1}\lambda_{2,1}+\theta_{2,2}$				
C1,3	$\lambda_{1,1}\Psi_{1,1}\lambda_{3,1}$	$\lambda_{2,1}\Psi_{1,1}\lambda_{3,1}$	$\lambda_{3,1}\Psi_{1,1}\lambda_{3,1}+\theta_{3,3}$			
C2,1	$\lambda_{1,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{2,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{3,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{4,2}\Psi_{2,2}\lambda_{4,2}+\theta_{4,4}$		
C2,2	$\lambda_{1,1}\Psi_{2,1}\lambda_{5,2}$	$\lambda_{2,1}\Psi_{2,1}\lambda_{5,2}$	$\lambda_{3,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{4,2}\Psi_{2,2}\lambda_{5,2}$	$\lambda_{5,2}\Psi_{2,2}\lambda_{5,2}+\theta_{5,5}$	
C2,3	$\lambda_{1,1}\Psi_{2,1}\lambda_{6,2}$	$\lambda_{2,1}\Psi_{2,1}\lambda_{6,2}$	$\lambda_{3,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{4,2}\Psi_{2,2}\lambda_{6,2}$	$\lambda_{5,2}\Psi_{2,2}\lambda_{6,2}$	$\lambda_{6,2}\Psi_{2,2}\lambda_{6,2}+\theta_{6,6}$



Covariance between “indicator 4” and “indicator 6” = $\lambda_{6,2}\Psi_{2,2}\lambda_{4,2}$

Σ = Model Implied Correlation/Covariance Matrix

	Indicator1	Indicator2	Indicator3	Indicator4	Indicator5	Indicator6
C1,1	$\lambda_{1,1}\Psi_{1,1}\lambda_{1,1}+\theta_{1,1}$					
C1,2	$\lambda_{1,1}\Psi_{1,1}\lambda_{2,1}$	$\lambda_{2,1}\Psi_{1,1}\lambda_{2,1}+\theta_{2,2}$				
C1,3	$\lambda_{1,1}\Psi_{1,1}\lambda_{3,1}$	$\lambda_{2,1}\Psi_{1,1}\lambda_{3,1}$	$\lambda_{3,1}\Psi_{1,1}\lambda_{3,1}+\theta_{3,3}$			
C2,1	$\lambda_{1,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{2,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{3,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{4,2}\Psi_{2,2}\lambda_{4,2}+\theta_{4,4}$		
C2,2	$\lambda_{1,1}\Psi_{2,1}\lambda_{5,2}$	$\lambda_{2,1}\Psi_{2,1}\lambda_{5,2}$	$\lambda_{3,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{4,2}\Psi_{2,2}\lambda_{5,2}$	$\lambda_{5,2}\Psi_{2,2}\lambda_{5,2}+\theta_{5,5}$	
C2,3	$\lambda_{1,1}\Psi_{2,1}\lambda_{6,2}$	$\lambda_{2,1}\Psi_{2,1}\lambda_{6,2}$	$\lambda_{3,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{4,2}\Psi_{2,2}\lambda_{6,2}$	$\lambda_{5,2}\Psi_{2,2}\lambda_{6,2}$	$\lambda_{6,2}\Psi_{2,2}\lambda_{6,2}+\theta_{6,6}$

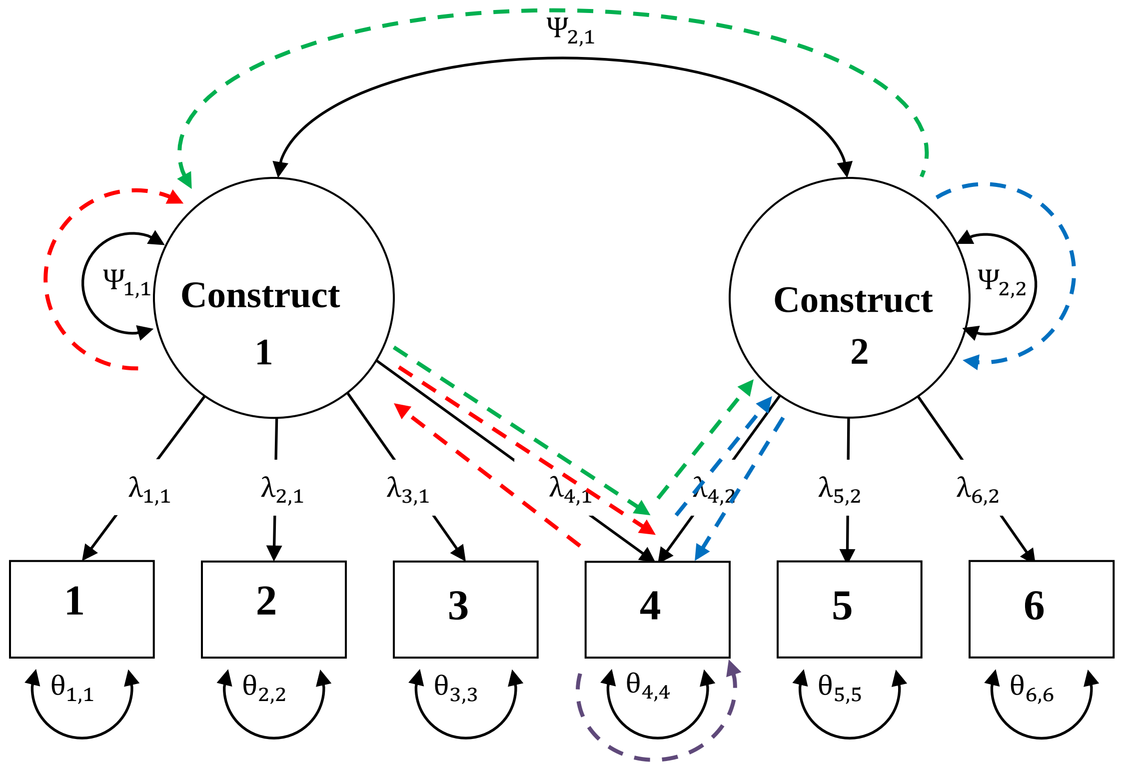


Covariance between “indicator 1” and “indicator 4” = $\lambda_{1,1}\Psi_{2,1}\lambda_{4,2}$

Σ = Model Implied Correlation/Covariance Matrix

	Indicator1	Indicator2	Indicator3	Indicator4	Indicator5	Indicator6
C1,1	$\lambda_{1,1}\Psi_{1,1}\lambda_{1,1}+\theta_{1,1}$					
C1,2	$\lambda_{1,1}\Psi_{1,1}\lambda_{2,1}$	$\lambda_{2,1}\Psi_{1,1}\lambda_{2,1}+\theta_{2,2}$				
C1,3	$\lambda_{1,1}\Psi_{1,1}\lambda_{3,1}$	$\lambda_{2,1}\Psi_{1,1}\lambda_{3,1}$	$\lambda_{3,1}\Psi_{1,1}\lambda_{3,1}+\theta_{3,3}$			
C2,1	$\lambda_{1,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{2,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{3,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{4,2}\Psi_{2,2}\lambda_{4,2}+\theta_{4,4}$		
C2,2	$\lambda_{1,1}\Psi_{2,1}\lambda_{5,2}$	$\lambda_{2,1}\Psi_{2,1}\lambda_{5,2}$	$\lambda_{3,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{4,2}\Psi_{2,2}\lambda_{5,2}$	$\lambda_{5,2}\Psi_{2,2}\lambda_{5,2}+\theta_{5,5}$	
C2,3	$\lambda_{1,1}\Psi_{2,1}\lambda_{6,2}$	$\lambda_{2,1}\Psi_{2,1}\lambda_{6,2}$	$\lambda_{3,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{4,2}\Psi_{2,2}\lambda_{6,2}$	$\lambda_{5,2}\Psi_{2,2}\lambda_{6,2}$	$\lambda_{6,2}\Psi_{2,2}\lambda_{6,2}+\theta_{6,6}$

Existence of a dual loading:

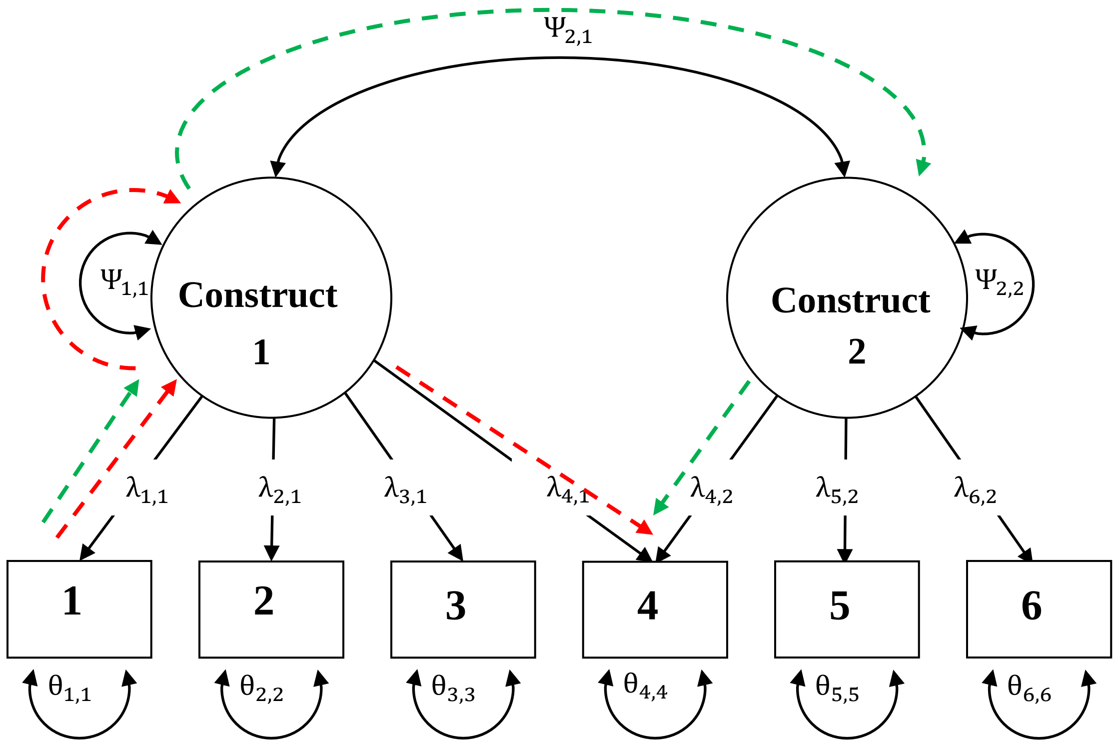


Variance of "indicator 4" =

$$\lambda_{4,1}\Psi_{1,1}\lambda_{4,1}+\lambda_{4,2}\Psi_{2,2}\lambda_{4,2}+2\lambda_{4,2}\Psi_{2,1}\lambda_{4,1}+\theta_{4,4}$$

Σ = Model Implied Correlation/Covariance Matrix

	Indicator 1	Indicator 2	Indicator 3	Indicator 4	Indicator 5	Indicator 6
C1,1	$\lambda_{1,1}\Psi_{1,1}\lambda_{1,1}+\theta_{1,1}$					
C1,2	$\lambda_{1,1}\Psi_{1,1}\lambda_{2,1}$	$\lambda_{2,1}\Psi_{1,1}\lambda_{2,1}+\theta_{2,2}$				
C1,3	$\lambda_{1,1}\Psi_{1,1}\lambda_{3,1}$	$\lambda_{2,1}\Psi_{1,1}\lambda_{3,1}$	$\lambda_{3,1}\Psi_{1,1}\lambda_{3,1}+\theta_{3,3}$			
C2,1	$\lambda_{1,1}\Psi_{2,1}\lambda_{4,2}+\lambda_{1,1}\Psi_{1,1}\lambda_{4,1}$	$\lambda_{2,1}\Psi_{2,1}\lambda_{4,2}+\lambda_{2,1}\Psi_{1,1}\lambda_{4,1}$	$\lambda_{3,1}\Psi_{2,1}\lambda_{4,2}+\lambda_{2,1}\Psi_{1,1}\lambda_{4,1}$	$\lambda_{4,2}^2\Psi_{2,2}+\lambda_{4,1}^2\Psi_{1,1}+2\lambda_{4,1}\Psi_{2,1}\lambda_{4,2}+\theta_{4,4}$		
C2,2	$\lambda_{1,1}\Psi_{2,1}\lambda_{5,2}$	$\lambda_{2,1}\Psi_{2,1}\lambda_{5,2}$	$\lambda_{3,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{14,2}\Psi_{2,2}\lambda_{5,2}$	$\lambda_{5,2}\Psi_{2,2}\lambda_{5,2}+\theta_{5,5}$	
C2,3	$\lambda_{1,1}\Psi_{2,1}\lambda_{6,2}$	$\lambda_{2,1}\Psi_{2,1}\lambda_{6,2}$	$\lambda_{3,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{14,2}\Psi_{2,2}\lambda_{6,2}$	$\lambda_{5,2}\Psi_{2,2}\lambda_{6,2}$	$\lambda_{6,2}\Psi_{2,2}\lambda_{6,2}+\theta_{6,6}$



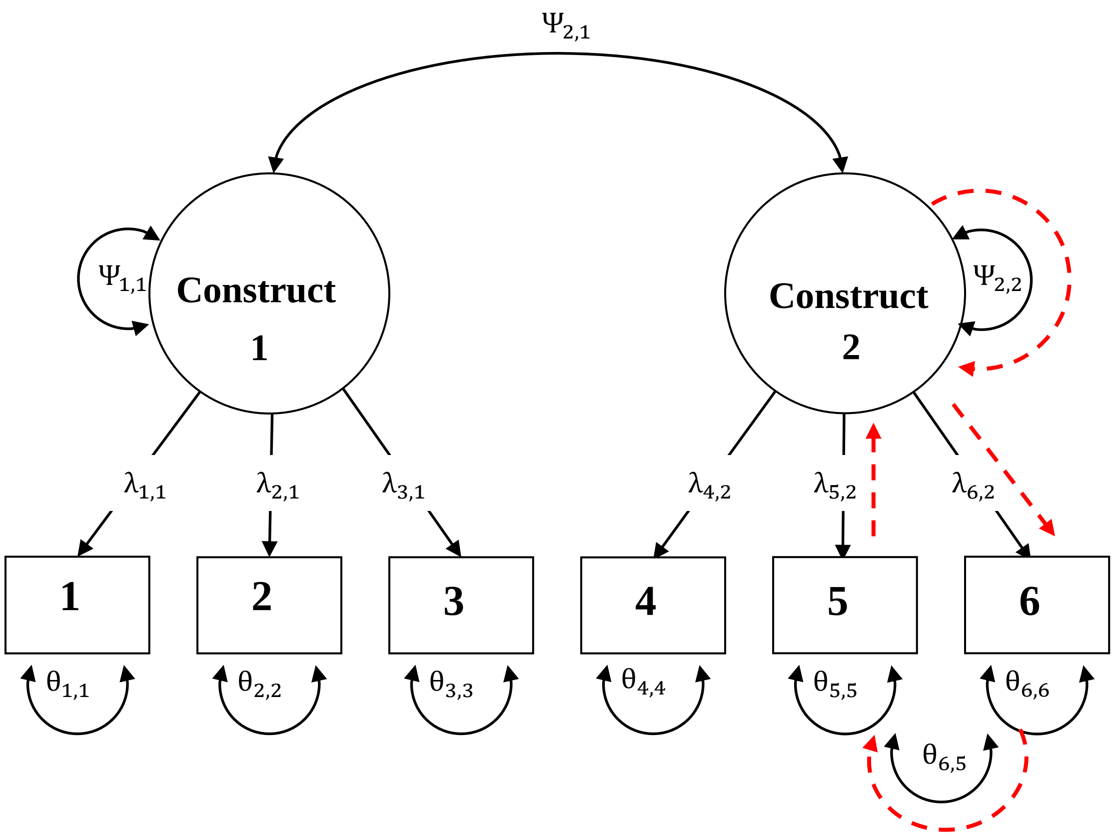
Covariance between "indicator 1" and "indicator 4" =

$$\lambda_{1,1}\Psi_{1,1}\lambda_{4,1}+\lambda_{1,1}\Psi_{2,1}\lambda_{4,2}$$

Σ = Model Implied Correlation/Covariance Matrix

	Indicator 1	Indicator 2	Indicator 3	Indicator4	Indicator 5	Indicator 6
C1,1	$\lambda_{1,1}\Psi_{1,1}\lambda_{1,1}+\theta_{1,1}$					
C1,2	$\lambda_{1,1}\Psi_{1,1}\lambda_{2,1}$	$\lambda_{2,1}\Psi_{1,1}\lambda_{2,1}+\theta_{2,2}$				
C1,3	$\lambda_{1,1}\Psi_{1,1}\lambda_{3,1}$	$\lambda_{2,1}\Psi_{1,1}\lambda_{3,1}$	$\lambda_{3,1}\Psi_{1,1}\lambda_{3,1}+\theta_{3,3}$			
C2,1	$\lambda_{1,1}\Psi_{2,1}\lambda_{4,2}+\lambda_{1,1}\Psi_{1,1}\lambda_{4,1}$	$\lambda_{2,1}\Psi_{2,1}\lambda_{4,2}+\lambda_{2,1}\Psi_{1,1}\lambda_{4,1}$	$\lambda_{3,1}\Psi_{2,1}\lambda_{4,2}+\lambda_{2,1}\Psi_{1,1}\lambda_{4,1}$	$\lambda_{4,2}^2\Psi_{2,2}+\lambda_{4,1}^2\Psi_{1,1}+2\lambda_{4,1}\Psi_{2,1}\lambda_{4,2}+\theta_{4,4}$		
C2,2	$\lambda_{1,1}\Psi_{2,1}\lambda_{5,2}$	$\lambda_{2,1}\Psi_{2,1}\lambda_{5,2}$	$\lambda_{3,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{14,2}\Psi_{2,2}\lambda_{5,2}$	$\lambda_{5,2}\Psi_{2,2}\lambda_{5,2}+\theta_{5,5}$	
C2,3	$\lambda_{1,1}\Psi_{2,1}\lambda_{6,2}$	$\lambda_{2,1}\Psi_{2,1}\lambda_{6,2}$	$\lambda_{3,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{14,2}\Psi_{2,2}\lambda_{6,2}$	$\lambda_{5,2}\Psi_{2,2}\lambda_{6,2}$	$\lambda_{6,2}\Psi_{2,2}\lambda_{6,2}+\theta_{6,6}$

Existence of a correlated residual:



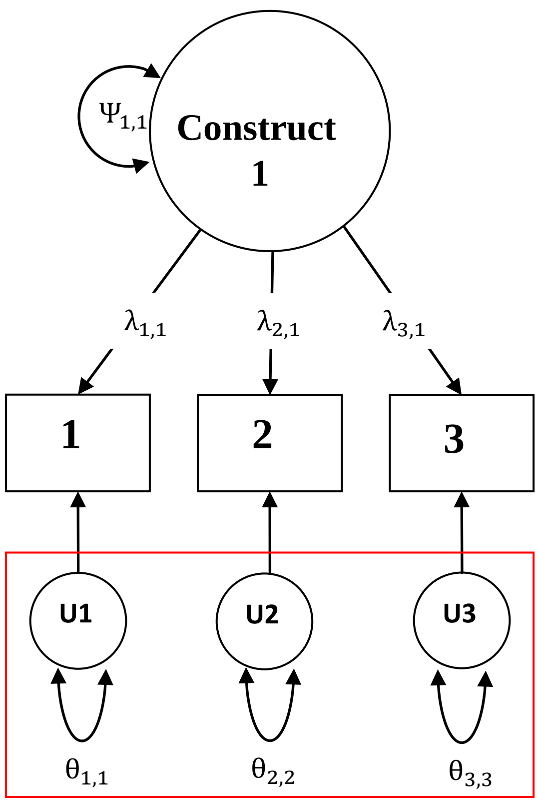
Covariance between "indicator 5" and "indicator 6" =

$$\lambda_{5,2}\Psi_{2,2}\lambda_{6,2} + \theta_{6,5}$$

Σ = Model Implied Correlation/Covariance Matrix

	Indicator1	Indicator2	Indicator3	Indicator4	Indicator5	Indicator6
C1,1	$\lambda_{1,1}\Psi_{1,1}\lambda_{1,1}+\theta_{1,1}$					
C1,2	$\lambda_{1,1}\Psi_{1,1}\lambda_{2,1}$	$\lambda_{2,1}\Psi_{1,1}\lambda_{2,1}+\theta_{2,2}$				
C1,3	$\lambda_{1,1}\Psi_{1,1}\lambda_{3,1}$	$\lambda_{2,1}\Psi_{1,1}\lambda_{3,1}$	$\lambda_{3,1}\Psi_{1,1}\lambda_{3,1}+\theta_{3,3}$			
C2,1	$\lambda_{1,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{2,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{3,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{4,2}\Psi_{2,2}\lambda_{4,2}+\theta_{4,4}$		
C2,2	$\lambda_{1,1}\Psi_{2,1}\lambda_{5,2}$	$\lambda_{2,1}\Psi_{2,1}\lambda_{5,2}$	$\lambda_{3,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{4,2}\Psi_{2,2}\lambda_{5,2}$	$\lambda_{5,2}\Psi_{2,2}\lambda_{5,2}+\theta_{5,5}$	
C2,3	$\lambda_{1,1}\Psi_{2,1}\lambda_{6,2}$	$\lambda_{2,1}\Psi_{2,1}\lambda_{6,2}$	$\lambda_{3,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{4,2}\Psi_{2,2}\lambda_{6,2}$	$\lambda_{5,2}\Psi_{2,2}\lambda_{6,2}+\theta_{6,5}$	$\lambda_{6,2}\Psi_{2,2}\lambda_{6,2}+\theta_{6,6}$

Convention 2:

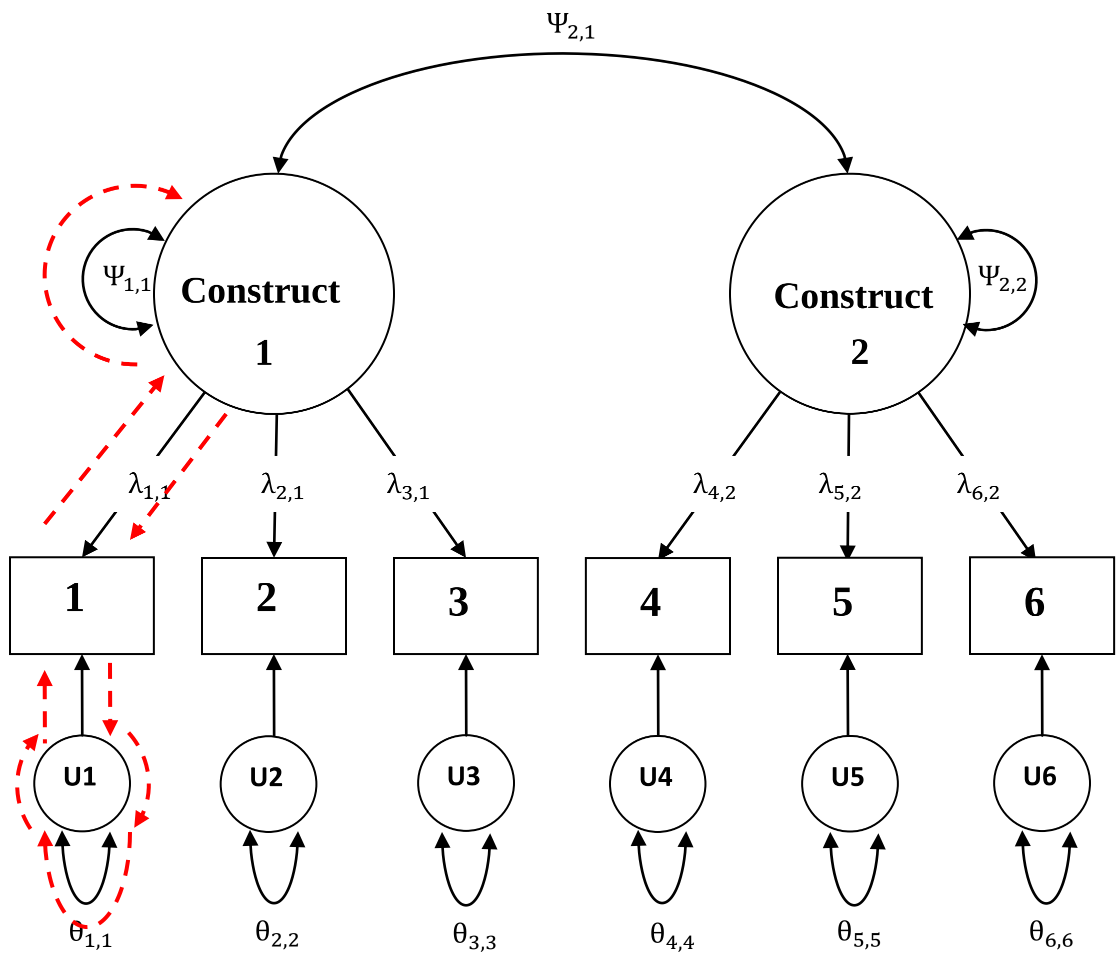


The technically accurate way: Residual-variance information is treated as a unique factor with some estimated variance.

Σ = Model Implied Correlation/Covariance Matrix *

	Indicator1	Indicator2	Indicator3	Indicator4	Indicator5	Indicator6
C1,1	$\lambda_{1,1}\Psi_{1,1}\lambda_{1,1}+\theta_{1,1}$					
C1,2	$\lambda_{1,1}\Psi_{1,1}\lambda_{2,1}$	$\lambda_{2,1}\Psi_{1,1}\lambda_{2,1}+\theta_{2,2}$				
C1,3	$\lambda_{1,1}\Psi_{1,1}\lambda_{3,1}$	$\lambda_{2,1}\Psi_{1,1}\lambda_{3,1}$	$\lambda_{3,1}\Psi_{1,1}\lambda_{3,1}+\theta_{3,3}$			
C2,1	$\lambda_{1,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{2,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{3,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{4,2}\Psi_{2,2}\lambda_{4,2}+\theta_{4,4}$		
C2,2	$\lambda_{1,1}\Psi_{2,1}\lambda_{5,2}$	$\lambda_{2,1}\Psi_{2,1}\lambda_{5,2}$	$\lambda_{3,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{4,2}\Psi_{2,2}\lambda_{5,2}$	$\lambda_{5,2}\Psi_{2,2}\lambda_{5,2}+\theta_{5,5}$	
C2,3	$\lambda_{1,1}\Psi_{2,1}\lambda_{6,2}$	$\lambda_{2,1}\Psi_{2,1}\lambda_{6,2}$	$\lambda_{3,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{4,2}\Psi_{2,2}\lambda_{6,2}$	$\lambda_{5,2}\Psi_{2,2}\lambda_{6,2}$	$\lambda_{6,2}\Psi_{2,2}\lambda_{6,2}+\theta_{6,6}$

*The corresponding path diagram is on the next page:

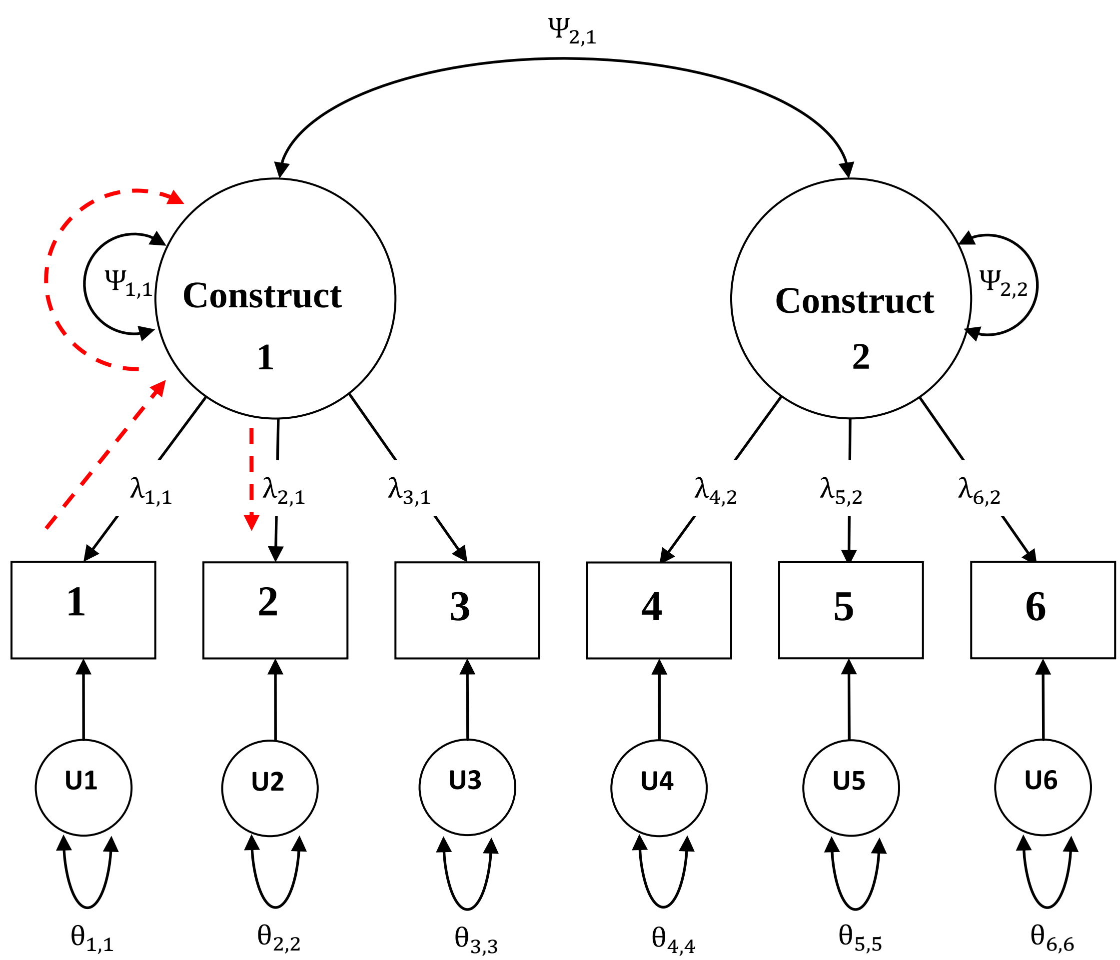


Variance of "indicator 1" = $\lambda_{1,1}\ \Psi_{1,1}\lambda_{1,1} + \theta_{1,1}$

<u>Σ = Model Implied Correlation/Covariance Matrix *</u>

	Indicator1	Indicator2	Indicator3	Indicator4	Indicator5	Indicator6
C1,1	$\lambda_{1,1}\Psi_{1,1}\lambda_{1,1}+\theta_{1,1}$					
C1,2	$\lambda_{1,1}\Psi_{1,1}\lambda_{2,1}$	$\lambda_{2,1}\Psi_{1,1}\lambda_{2,1}+\theta_{2,2}$				
C1,3	$\lambda_{1,1}\Psi_{1,1}\lambda_{3,1}$	$\lambda_{2,1}\Psi_{1,1}\lambda_{3,1}$	$\lambda_{3,1}\Psi_{1,1}\lambda_{3,1}+\theta_{3,3}$			
C2,1	$\lambda_{1,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{2,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{3,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{4,2}\Psi_{2,2}\lambda_{4,2}+\theta_{4,4}$		
C2,2	$\lambda_{1,1}\Psi_{2,1}\lambda_{5,2}$	$\lambda_{2,1}\Psi_{2,1}\lambda_{5,2}$	$\lambda_{3,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{4,2}\Psi_{2,2}\lambda_{5,2}$	$\lambda_{5,2}\Psi_{2,2}\lambda_{5,2}+\theta_{5,5}$	
C2,3	$\lambda_{1,1}\Psi_{2,1}\lambda_{6,2}$	$\lambda_{2,1}\Psi_{2,1}\lambda_{6,2}$	$\lambda_{3,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{4,2}\Psi_{2,2}\lambda_{6,2}$	$\lambda_{5,2}\Psi_{2,2}\lambda_{6,2}$	$\lambda_{6,2}\Psi_{2,2}\lambda_{6,2}+\theta_{6,6}$

*The corresponding path diagram is on the next page:

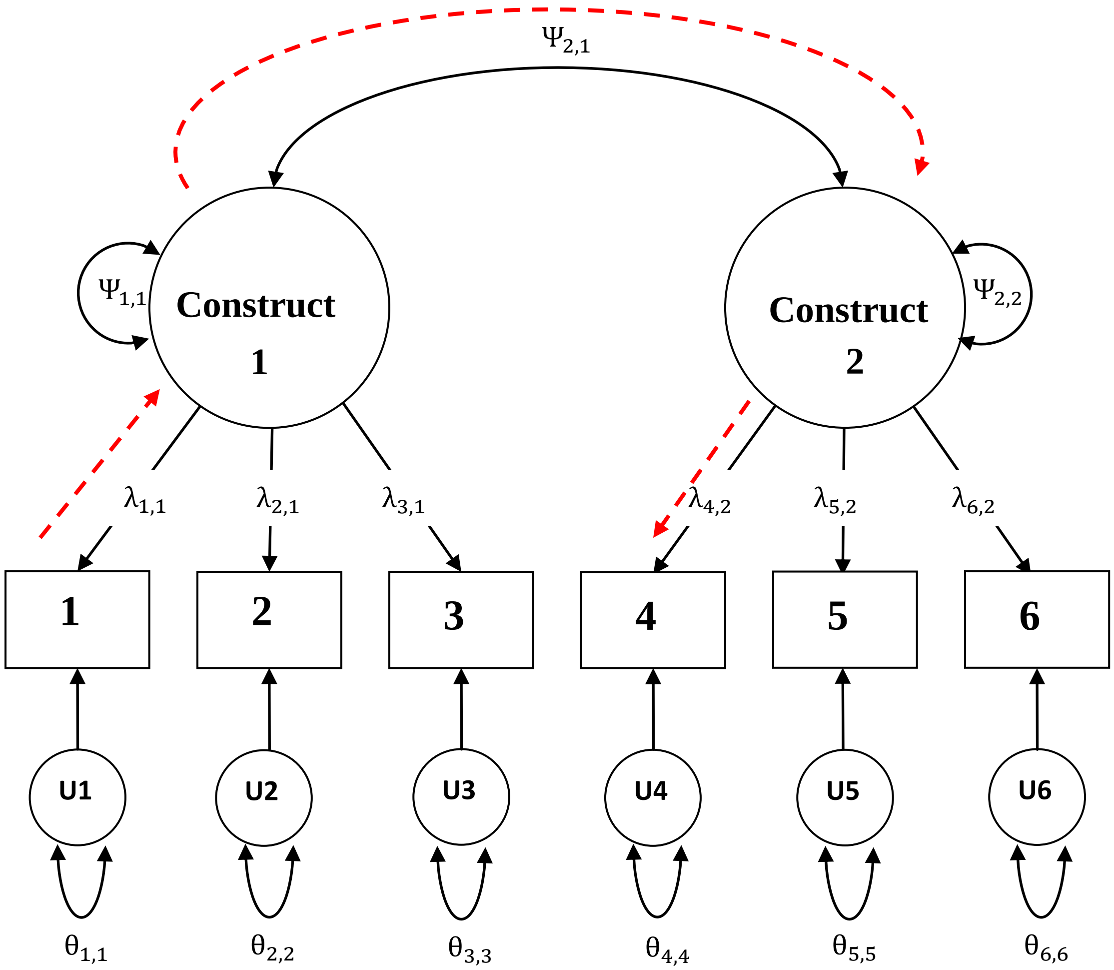


Covariance between "indicator 1" and "indicator 2" = $\lambda_{1,1}\Psi_{1,1}\lambda_{2,1}$

Σ = Model Implied Correlation/Covariance Matrix *

	Indicator1	Indicator2	Indicator3	Indicator4	Indicator5	Indicator6
C1,1	$\lambda_{1,1}\Psi_{1,1}\lambda_{1,1}+\theta_{1,1}$					
C1,2	$\lambda_{1,1}\Psi_{1,1}\lambda_{2,1}$	$\lambda_{2,1}\Psi_{1,1}\lambda_{2,1}+\theta_{2,2}$				
C1,3	$\lambda_{1,1}\Psi_{1,1}\lambda_{3,1}$	$\lambda_{2,1}\Psi_{1,1}\lambda_{3,1}$	$\lambda_{3,1}\Psi_{1,1}\lambda_{3,1}+\theta_{3,3}$			
C2,1	$\lambda_{1,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{2,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{3,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{4,2}\Psi_{2,2}\lambda_{4,2}+\theta_{4,4}$		
C2,2	$\lambda_{1,1}\Psi_{2,1}\lambda_{5,2}$	$\lambda_{2,1}\Psi_{2,1}\lambda_{5,2}$	$\lambda_{3,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{4,2}\Psi_{2,2}\lambda_{5,2}$	$\lambda_{5,2}\Psi_{2,2}\lambda_{5,2}+\theta_{5,5}$	
C2,3	$\lambda_{1,1}\Psi_{2,1}\lambda_{6,2}$	$\lambda_{2,1}\Psi_{2,1}\lambda_{6,2}$	$\lambda_{3,1}\Psi_{2,1}\lambda_{4,2}$	$\lambda_{4,2}\Psi_{2,2}\lambda_{6,2}$	$\lambda_{5,2}\Psi_{2,2}\lambda_{6,2}$	$\lambda_{6,2}\Psi_{2,2}\lambda_{6,2}+\theta_{6,6}$

*The corresponding path diagram is on the next page:



Covariance between "indicator 1" and "indicator 4" = $\lambda_{1,1}\Psi_{2,1}\lambda_{4,2}$

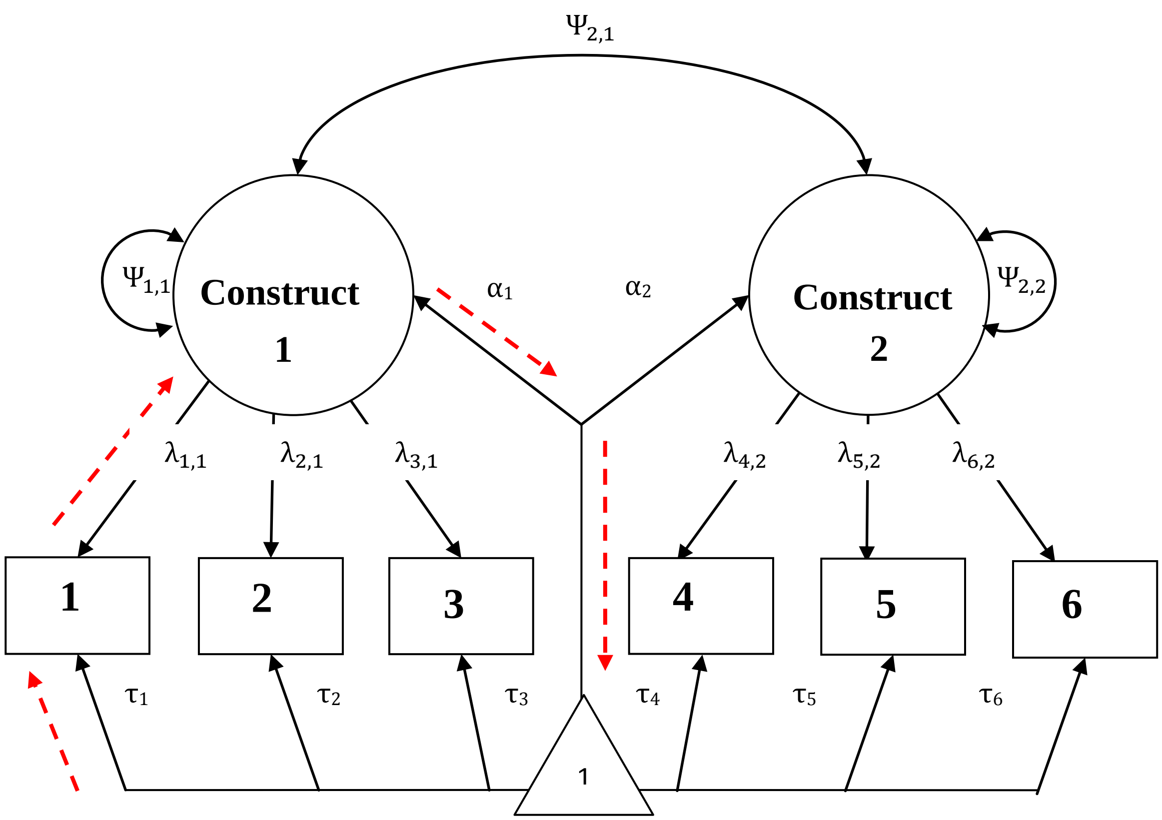
Tracing Rules for Means

- Indicator means consist of a common factor component and a unique component (intercept)
- Observed mean is found by multiplicatively tracing all paths from one indicator to an intercept/mean (i.e., the *constant*) then adding all values
  - Can only go through unidirectional arrows
  - Can only go "up" an arrow

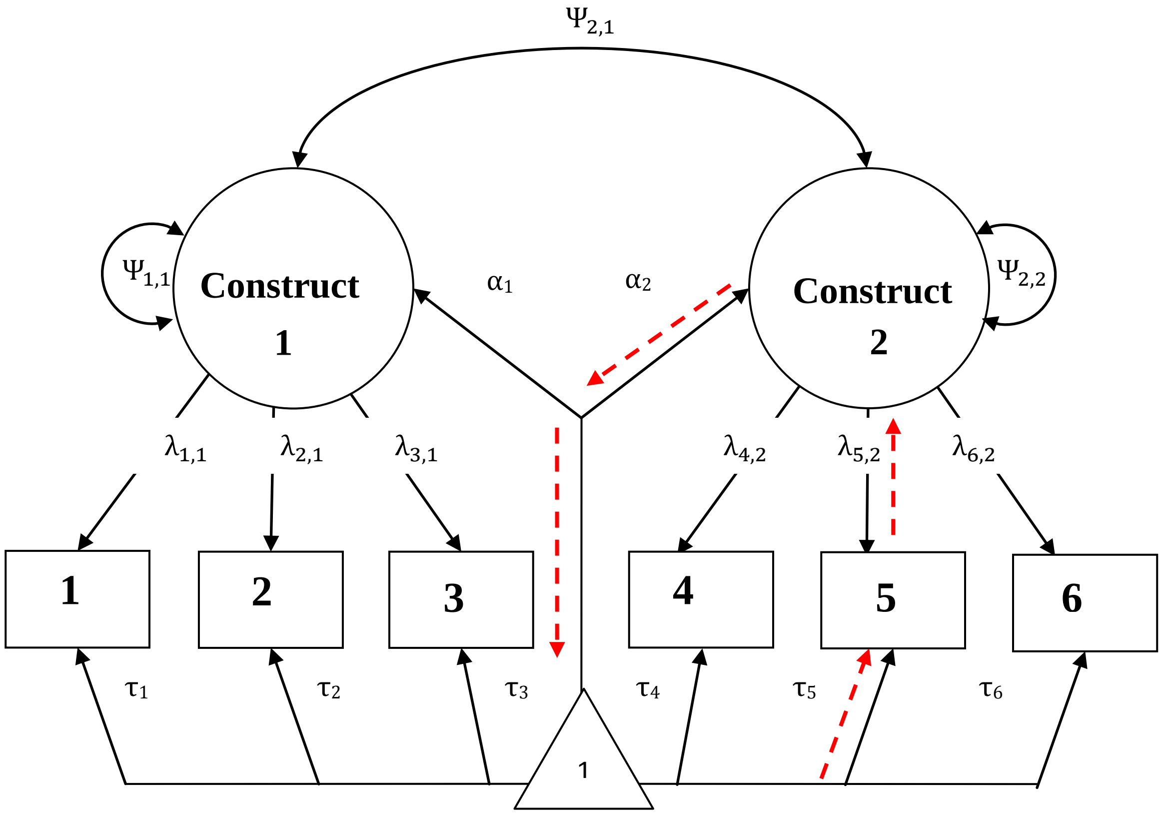
Formula to Reproduce Means:

- Indicator Mean = Intercept + Loading*(Latent Mean)
- $\overline{Y} \approx \tau + \lambda(\alpha) = \mu$

Path Diagram Illustration:



Mean of "indicator 1" = $\tau_1 + \lambda_{1,1}(\alpha_1)$



Mean of “indicator 5” = $\tau_5 + \lambda_{5,2}(\alpha_2)$

Reference

Wright, S. (1934). The method of path coefficients. *The Annals of Mathematical Statistics, 5*(3), 161-215.