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*Teaching Mathematics to Middle School Students with Learning Disabilities*
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CHAPTER 5

# Self-Regulation Strategies for Better Math Performance in Middle School

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Mr. Sosa teaches four classes of seventh-grade general mathematics and one remedial math class. The students in these classes represent diverse abilities and achievement levels. In addition to the remedial class, he has many students in the general education classes who have considerable difficulty in mathematics, especially in solving mathematical word problems. Many of the students in his classes have identified learning disabilities, a few have been identified with attention-deficit/hyperactivity disorder, and still others are second-language learners. Other students of Mr. Sosa's, however, can solve typical textbook problems like the following because they have mastered the problem-solving strategies needed to be successful. As important, they have developed a variety of self-regulation strategies that help them monitor and evaluate their problem solving. The challenge for Mr. Sosa is to teach the students who are having difficulty solving problems the strategies that the other students use effectively and efficiently.

These are typical textbook math problems Mr. Sosa's good problem solvers are able to solve with relative ease:

> A train going to New York travels 75 miles per hour for 1 hour. Then, because of weather problems, it slows to 35 miles per hour for the rest of the trip. If the trip takes 8 hours, how many miles has the train traveled when it gets to New York?

> Mr. Hanson bought a used car for $5,000. His monthly payment was $173.32 for 3 years. What is the amount of interest he was charged?

Many students, especially those with learning disabilities (LD), have considerable difficulty solving problems like these. They may have the basic computational and procedural knowledge and skills needed but still cannot solve them. The purpose of this chapter is to explain why students have so much difficulty and what teachers like Mr. Sosa can do to help students become better math problem solvers. The following three questions frame the chapter.

- Why are students with LD such poor mathematical problem solvers?
- What do good problem solvers do to solve math problems?
- How can we teach students with LD to be better math problem solvers?

Examples of students' problem-solving and instructional vignettes are provided to guide teachers. Additionally, Solve It!, a math-problem-solving instructional program validated with middle school students with LD, is described (Montague, 2003).

## WHY ARE STUDENTS WITH LEARNING DISABILITIES SUCH POOR MATHEMATICAL PROBLEM SOLVERS?

Many students with LD have serious perceptual, memory, language, and/or reasoning problems that interfere with mathematical problem solving (Bley & Thornton, 1995). That is, students may have trouble reading and understanding the problem, attending to the information in the problem, identifying important information and representing that information, developing a plan to solve the problem, and computing (e.g., recalling math facts and remembering algorithmic procedures). Even though students may have acquired the basic knowledge and skills in reading and mathematics and, therefore, should be able to carry out these cognitive activities, they often do not because of these problems. Additionally, these students often experience significant self-regulation problems that interfere with problem solving.

Students with LD characteristically are deficient in the ability to select appropriate strategies to use and to regulate themselves during academic tasks (Wong, Harris, Graham, & Butler, 2003). That is, they have self-regulation problems that prevent successful completion of tasks. These students are typically disorganized, do not know where or how to begin, lack enabling strategies, and do not evaluate what they do. The ability to regulate one's cognitive activities underlies the executive processes associated with metacognition (Flavell, 1976). Metacognition consists of both *knowledge and awareness* of one's cognitive strengths and weaknesses and *self-regulation*, the ability to coordinate that awareness with appropriate action (Wong, 1999). Metacognition develops in young children from an early age and matures during early adolescence, sometime between the ages of 11 and 14. Metacognitive ability is essential for successful academic performance across domains (Montague, 1998).

For mathematical problem solving, students need to be able to determine if they understand the problem after they read it, recognize the important information, develop a visual representation of the problem that reflects the important information, make a logical plan to solve the problem, think about a reasonable solution and answer, compute with confidence, and verify their solution as accurate. They need to be able to guide themselves through the process as they execute the solution by using self-regulation strategies. These strategies include self-verbalization, self-questioning, and self-evaluation. In other words, students need to be able to tell themselves what to do, ask themselves questions to determine if they have acted appropriately, monitor their performance as they solve the problem, and, finally, check and verify that what they have done is correct.

To illustrate, take a moment to solve the following problem.

> Caroline owns a dog kennel. She usually has 15 dogs to care for every week. Each dog eats about 10 pounds of food per week. She pays $1.60 per pound for the food. How much does Caroline pay to feed 15 dogs each week?

Now, stop and make a list of the cognitive processes and metacognitive strategies you used to solve the problems. Most people engage in some or all of the following activities, depending on the difficulty level of the problem:

- Rereading the problem or parts of the problem
- Identifying the important information
- Asking themselves questions
- Putting the problem into their own words
- Visualizing or drawing a picture or diagram of the problem
- Telling themselves what to do
- Making a plan
- Estimating the outcome
- Working backward and forward
- Checking that the process and the product are correct

To reiterate, students with LD generally are poor problem solvers due to strategy deficits or differences that impede effective and efficient problem solving. They may have a repertoire of strategies and yet have difficulty selecting appropriate strategies and organizing and executing them. They also are inefficient in abandoning and replacing ineffective strategies, do not readily adapt previously used strategies, and do not generalize strategy use. Students with LD need help in acquiring and applying cognitive processes and self-regulation strategies that underlie effective and efficient problem solving. For math problem solving, they need to learn how to understand the mathematical problems, analyze the information presented, develop logical plans to solve problems, and evaluate their solutions.

## HOW DO GOOD PROBLEM SOLVERS SOLVE MATH PROBLEMS?

We know that good problem solvers are good strategic learners and that students with LD are poor strategic learners. There are several other characteristics that differentiate good and poor problem solvers. Good problem solvers usually are highly motivated and persist in their effort. They control their emotions and are appropriately confident. They focus their attention appropriately and are self-directed and self-regulating. Poor problem solvers, on the other hand, have low motivation and give up easily. They lack strategies or have a limited repertoire, and if they have acquired strategies, they experience difficulty selecting, organizing, and using them appropriately. They are poor self-regulators and are unable to detect and correct errors. Table 5.1 lists the salient differences between good and poor problem solvers.

We investigated the math problem-solving processes and strategies of middle school students by having students "think aloud" while they solved problems (Montague & Applegate, 1993). The example below shows how Ana, an average-achieving eighth-grade student, used self-regulation strategies to guide her as she solved the following problem.

> Four friends have decided they want to go to the movies on Saturday. Tickets are $2.75 for students. Altogether they have $8.40. How much more do they need?

### Ana's Think-Aloud

"OK, first I am going to read it to make sure. [She reads the problem.] I will read it twice to make sure that I understand it. [She reads the problem again.] Then I am going to pick out the numbers and see if they are neces-

**TABLE 5.1. Differences between Good and Poor Problem Solving and Strategic Learning**

| Good | Poor |
|---|---|
| • Repertoire of strategies | • Limited strategies |
| • Metacognitive approach | • Immature metacognitive abilities |
| • Motivated | • Low motivation |
| • Memory capacity | • Attention, memory, language problems |
| • Developed language | • Impulsive |
| • Appropriately confident | • Uncertain approach to problems |
| • Attentional focus | • Inability to detect and correct errors |
| • Self-directed and self-regulating | • Problem representation difficulties |
| • Ability to generalize learning | • Poor generalizers |

> sary. Four friends have decided they want to go to the movies on Saturday. Tickets are $2.75 for students. So, we have two numbers already, 4 and $2.75, so I am multiplying to get the answer of how much money all the tickets are going to cost for all the friends—$2.75 times 4. [She computes.] So, then you already have your answer. Now it says altogether they have $8.40. So now you have to subtract to see how much more money they need—11 minus 8.40. Oh, let's see. [She computes.] I always check my work by going back and adding to see if it's right, the subtraction, because sometimes I have a bit of trouble so I go back. [She checks her computation.] That's it. I'm done."

Ana clearly uses self-regulation strategies. She tells herself what to do as she progresses through the problem, breaks the problem into parts, identifies the important information, notes the question, and assures herself that she understands it by reading and rereading and making a plan. She monitors her performance by talking herself through the problem and checking that she completes each step correctly. Eric is another average-achieving student. Let's look at his solution to a different problem.

> A group bought 52 airline tickets. Each ticket was $26 less than the $280 regular-price ticket. How much did the group spend for the tickets?

## Eric's Think-Aloud

> "[He reads the problem.] Somewhat easy. A group bought 52 tickets. I am going to write 52. Each ticket was $26 less than the $280 regular price ticket. How much did the group spend on tickets? So, I am going to look back at the problem and so I am going to multiply 52 times 26. [He computes.] Then it's no less than . . . OK, I don't know what I just did. I really don't know. OK, each ticket is $26 less than the regular-price ticket. Why is it a $280 regular-price ticket? OK, I am going to do this one over again. A group bought 52 tickets. Each ticket was $26 less than the $280 regular-price ticket. So the tickets used to cost $280. OK. So, how much did the group spend for tickets? Oh, OK, that's easy. First I need to subtract 26 from 280. Then I multiply that number by 52. [He computes.] Now I understand what I did."

Like Ana, Eric uses self-regulation strategies to guide himself as he solves the problem. First, he evaluates the difficulty level and decides the problem is "somewhat easy." He tells himself what to do and asks himself questions. He monitors his performance and realizes that he does not understand what he did about halfway through and decides to start over by reading the problem again. He makes sure he understands the problem and clearly sets a plan. When he finishes, he acknowledges his understanding and is satisfied. Now look at Greg's think-aloud for the same problem. Greg, an eighth-grader, was placed in a learning disabilities program when he was in the fourth grade.

### Greg's Think-Aloud

> "[He reads the problem.] 280 take away 26 is 6, 8 take away 2 is 6, and 2 is 266."

Greg seemingly has no strategies in place for solving the problem. Presumably, he sees the word *less* and subtracts without any clear understanding of the problem. Greg is typical of most students with LD who have no resources for problem solving. He needs explicit instruction in how to read, understand, analyze, and evaluate math problems and, most notably, needs instruction in self-regulation strategies. The primary self-regulation strategies throughout the process of math problem solving are self-instruction, or telling yourself what to do, self-questioning, or asking yourself questions, and self-monitoring, or checking yourself. Self-regulation strategies help students gain access to the content of problem solving (i.e., the cognitive processes and strategies that good problem solvers use), apply those processes and strategies, and regulate their use of processes and strategies as well as their overall performance as they solve problems.

Good problem solvers use a variety of processes and strategies as they read and represent the problem before they make a plan to solve it. First, they *read* the problem for understanding. As they read, they use comprehension strategies to translate the linguistic and numerical information in the problem into mathematical notations. For example, good problem solvers may read the problem more than once and may reread parts of the problem as they progress and think through it. They use self-regulation strategies by asking themselves if they understood the problem and by monitoring their performance as they solve the problem.

They *paraphrase* the problem by putting it into their own words. They identify the important information and may even underline parts of the problem. Good problem solvers ask themselves what the question is and what they are looking for. They check the information against the problem and the question. *Visualizing* or drawing a picture or diagram means developing a schematic representation of the problem so that the picture or image reflects the relationships among all the important problem parts. Using both verbal translation and visual representation, good problem solvers not only are guided toward understanding the problem, but they are also guided toward developing a plan to solve the problem. Here is the point at which students decide what to do to solve the problem. They tell themselves to make a drawing or develop a visual representation that shows the relationships among the problem parts. They check the "picture" against the problem information. They have represented the problem and they are now ready to develop a solution path.

They *hypothesize* by thinking about logical solutions and the types of operations and number of steps needed to solve the problem. They may write the operation symbols as they decide on the most appropriate solution path and the algorithms they need to carry out the plan. They tell themselves to decide what steps and operations are needed. They ask themselves if the plan makes sense given the information they have and monitor themselves to ensure that the plan is a good one as they continue. Good problem solvers usually *estimate* or predict the answer using mental calculations or even may quickly use paper and pencil as they round the numbers up and down to get a "ballpark" idea. They tell themselves to round the numbers both up and down and ask themselves if they did. They check that they used all the important information.

They are now ready to *compute*. So they tell themselves to do the arithmetic and then compare their answer with their estimate. They also ask themselves if the answer makes sense and if they have used all the necessary symbols and labels such as dollar signs and decimals. They check to make sure that all the operations were done in the right order and that they followed their plan. Finally, they *check* to make sure they used the correct procedures and that their answer is correct. They check the plan and the computation. They ask themselves if they have checked every step and if they computed correctly. They ask if their answer is accurate, and, if they are unsure, they ask for help.

Students who are poor mathematical problem solvers, as most students with LD are, do not process problem information effectively or efficiently. They lack or do not apply the resources needed to complete this complex cognitive activity. These students also lack the self-regulation strategies that good problem solvers use. To help students with LD to become good problem solvers, teachers must understand and teach the cognitive processes and self-regulation strategies that good problem solvers use. That is, they must teach the content of math-problem-solving instruction. To do this, they must use instructional procedures that are research based and have proven effectiveness. These procedures are the basis of cognitive strategy instruction, which has been demonstrated to be one of the most powerful interventions for students with LD (Swanson & Hoskyn, 2001). Cognitive strategy instruction is characterized by an instructional routine that emphasizes guided discussion and interactive activities, verbal rehearsal of processes and self-regulation strategies, active engagement in the learning process, student commitment to performance goals, acquisition and application of cognitive processes and strategies, practice and mastery, progress monitoring, and immediate success.

The content of math-problem-solving instruction is the host of cognitive processes and self-regulation strategies that good problem solvers use to solve mathematical problems. Students must learn how to use these processes and strategies not only effectively but efficiently as well. Figure 5.1 lists the processes and their accompanying self-regulation strategies that facilitate application of the processes (Montague, 2003).

Teaching self-regulation strategies as a component of cognitive strategy instruction helps students to take control of their actions, make appropriate decisions, and become independent problem solvers. These strategies facilitate math problem solving by having students tell themselves what to do (self-instruction), ask themselves questions as they go about solving problems (self-questioning), and

**READ** (for understanding)

**Say:** Read the problem. If I don't understand, read it again.

**Ask:** Have I read and understood the problem?

**Check:** For understanding as I solve the problem.

**PARAPHRASE** (your own words)

**Say:** Underline the important information. Put the problem in my own words.

**Ask:** Have I underlined the important information? What is the question? What am I looking for?

**Check:** That the information goes with the question.

**VISUALIZE** (a picture or a diagram)

**Say:** Make a drawing or a diagram. Show the relationships among the problem parts.

**Ask:** Does the picture fit the problem? Did I show the relationships?

**Check:** The picture against the problem information.

**HYPOTHESIZE** (a plan to solve the problem)

**Say:** Decide how many steps and operations are needed. Write the operation symbols (+, −, ×, and /).

**Ask:** If I . . ., what will I get? If I . . ., then what do I need to do next? How many steps are needed?

**Check:** That the plan makes sense.

**ESTIMATE** (predict the answer)

**Say:** Round the numbers, do the problem in my head, and write the estimate.

**Ask:** Did I round up and down? Did I write the estimate?

**Check:** That I used the important information.

**COMPUTE** (do the arithmetic)

**Say:** Do the operations in the right order.

**Ask:** How does my answer compare with my estimate? Does my answer make sense? Are the decimals or money signs in the right places?

**Check:** That all the operations were done in the right order.

**CHECK** (make sure everything is right)

**Say:** Check the plan to make sure it is right. Check the computation.

**Ask:** Have I checked every step? Have I checked the computation? Is my answer right?

**Check:** That everything is right. If not, go back. Ask for help if I need it.

**FIGURE 5.1.** Math problem-solving processes and strategies.

check themselves throughout the problem-solving process (self-checking). Self-instruction involves providing one's own prompts and talking oneself through the problem-solving routine. Students may initially have difficulty using self-instruction because they may have difficulty verbalizing and remembering sequences of behaviors or activities. Self-instruction combined with self-questioning can be even more effective. Self-questioning is a form of cognitive cueing that helps remind students to use certain processes, skills, and behaviors. Students need to be taught which questions to ask and how to ask those questions as they solve problems. For example, after paraphrasing the problem, they should ask themselves, "Have I underlined the important information? What is the question? What am I looking for?"

Self-checking is used to help students reflect on the problem to make sure they selected an appropriate solution path and that they did not make any computational or procedural mistakes. The cognitive processes dictate the self-checking responses. Students learn to check:

- That they understand the problem
- That the information goes with the problem
- That the schematic representation reflects the problem information and shows the relationships among the problem parts
- That the plan makes sense
- That they used all the important information
- That the operations were completed in the right order
- That the answer is accurate. If not, they tell themselves to return to the problem and, if they still experience difficulty, to ask for help. Students need to be taught how to determine if they need help, whom to ask, and how to ask for it.

## HOW CAN WE TEACH STUDENTS WITH LEARNING DISABILITIES TO BE BETTER MATH PROBLEM SOLVERS?

The cornerstone of cognitive strategy instruction is explicit instruction (for the application for mathematical problem solving, see Montague, Warger, & Morgan, 2000). Explicit instruction incorporates research-based practices and instructional procedures such as cueing, modeling, verbal rehearsal, and feedback. The lessons are highly organized and structured. Appropriate cues and prompts are given as students learn and practice the cognitive processes and self-regulation strategies. Students are given individualized, immediate, corrective, and positive feedback on performance. Instruction stresses overlearning, mastery, and automaticity. Students are active participants as they learn and practice math-problem-solving processes and strategies and interact with other students and their teachers.

A guided discussion technique is used to promote active teaching and learning. Students are engaged from the outset, beginning with a discussion about why mathematical problem solving is important. Students take a baseline measure to determine their individual performance level. This baseline provides the foundation for setting individual performance goals. With the teacher, students set individual performance goals and make a commitment to becoming better problem solvers. The problem-solving activities embedded in cognitive strategy instruction are described next. The sample lesson at the end of the chapter is an actual problem-solving demonstration by a mathematics teacher. The teacher "thinks aloud" while solving a problem to demonstrate how good problem solvers approach and solve a problem. The lesson illustrates how problem solving is modeled for students when the cognitive processes and metacognitive strategies are introduced.

## Verbal Rehearsal

Before students begin to solve math problems, they must first memorize the cognitive processes and self-regulation strategies necessary for math problem solving. This content is introduced and demonstrated by the teacher to provide a context for application of the processes and strategies. Verbal rehearsal is a mnemonic strategy that enables students to memorize and recall automatically the labels and definitions of the math-problem-solving processes and strategies (Smith, 1998). Frequently, acronyms are created to help students remember as they verbally rehearse and internalize the labels and definitions for the processes and strategies. For math problem solving, the acronym RPV-HECC was created:

- **R** = Read for understanding
- **P** = Paraphrase—in your own words
- **V** = Visualize—draw a picture or diagram
- **H** = Hypothesize—make a plan
- **E** = Estimate—predict the answer
- **C** = Compute—do the arithmetic
- **C** = Check—make sure everything is right.

Cues and prompts are used initially to help students as they memorize the processes and their definitions. The goal is for students to recite from memory all processes and name the corresponding self-regulation strategies (the say, ask, check sequence for each process). When students have memorized the processes and are familiar with the self-regulation strategies for math problem solving, they can cue other students and the teacher as they begin to use the processes and strategies to solve problems.

## Process Modeling

Process modeling, sometimes referred to as cognitive modeling, is simply thinking aloud while demonstrating an activity. Process modeling has been shown to enhance reading comprehension, computation skills, question asking and answering behavior, problem solving, and other academic and social behaviors (e.g., Montague, Applegate, & Marquard, 1993). For mathematical problem solving, this means that the problem solver says everything he or she is thinking and doing while solving a math problem. When students are first learning how to apply the processes and strategies, the teacher demonstrates and models what good problem solvers do as they solve problems. Students have the opportunity to observe and hear how good problem solvers solve mathematical problems. Both correct and incorrect problem-solving behaviors are modeled. Modeling of correct behaviors helps students understand how good problem solvers use the processes and strategies appropriately. Modeling of incorrect behaviors allows students to learn how to use self-regulation strategies to monitor their performance and locate and correct errors. Self-regulation strategies are learned and practiced in the actual context of problem solving. When students learn the problem-solving routine and can apply it, they then exchange places with the teacher and become models for their peers.

Initially, students will need plenty of prompting and reinforcement as they become more comfortable with the problem-solving routine. However, they soon become proficient and independent in demonstrating how good problem solvers solve math problems. One of the instructional goals is to gradually move students from overt to covert verbalization. As students become more effective problem solvers, they will begin to verbalize covertly and then internally. In this way, they not only become more effective problem solvers, but they also become more efficient problem solvers. As students become more adept at problem solving, they will begin to adapt and modify the processes and strategies and "make them their own." That is, they become better at evaluating the difficulty level of problems and, as a result, become more efficient, or "faster and better."

## Visualization

Visualization, critical to problem representation, is the basis for understanding the problem (van Garderen & Montague, 2003). Visualization enables students to construct an image of the problem mentally or on paper . Students with LD are notoriously poor at visualization and, therefore, must be shown how to select the important information in the problem and develop a schematic representation. A schematic representation shows the relationships among the problem parts. Teachers must model how to draw a picture or make a diagram that shows the relationships among the problem parts using both the linguistic and numerical information in the problem. Visual representations can take many forms and will vary from student to student. Students may use a variety of visual representations such as pictures, tables, graphs, or other graphic displays. However, simply drawing a picture is insufficient. The graphic display or mental image must reflect the relationships among the pieces of information in the problem. Initially, students must be told to use paper and pencil because this is a new way of approaching math problems; later, as they become more proficient problem solvers, they will progress to mental images. Interestingly, if good problem solvers decide the problem is novel or challenging, they typically return to conscious application of processes and strategies.

## Role Reversal

Students with LD tend to be dependent rather than independent learners. One instructional procedure that promotes independent learning is role reversal. As students become comfortable with the math-problem-solving routine, they can "change places" with the teacher; that is, they can assume the role of the teacher as model and expert. An overhead projector is preferable to chalkboards for demonstrations because it allows the problem solver to face the group and interact more directly. The students can then engage in process modeling just as the teacher did to demonstrate that they can apply effectively the cognitive processes and self-regulation strategies they have learned. Other students can prompt or ask questions for clarification. In this way, students learn to think about, explain, and justify their visual representations and their solution paths. Teachers may also take the role of the student who then guides the "student as teacher" through the process. This interaction allows students to appreciate that there is usually more than one correct solution path for a math problem; that is, problems can be solved in a variety of ways.

## Peer Coaching

Peer coaching (i.e., peer partners, teams, and small problem-solving groups) gives students opportunities to see how other students approach mathematical problems differently, how they use cognitive processes and self-regulation strategies differently, and how they represent and solve problems differently. Peer coaching is a very effective instructional practice (Jenkins & O'Connor, 2003). Students gain a broader perspective on the problem-solving process and begin to realize that there is more than one way to solve a problem. As a result, they become more flexible and tolerant as thinkers. With partners or as a member of a group, students are supported and encouraged as they discuss the problems. They work cooperatively toward common solutions while appreciating the differences in approaches to each problem. They have ample opportunity to explain and clarify their choices. When students reach their performance goals and demonstrate mastery, novel or "real-life" problems like the following can be introduced for the partners, teams, or small groups (Montague, 2003).

> **Novel Mathematical Problem for Partner, Team, or Group Problem Solving**
>
> Your parents want to buy new school clothes for you, and they said you could spend $150. Make a list of items you would like to buy. Use newspaper ads to find prices. Then, decide which items you will

actually purchase. Work with your group to complete your list. Compare your final purchases with the purchases of the other group members.

## Performance Feedback

One of the most important instructional procedures is performance feedback (Swanson, 1999). It is critical to the success of the instructional program. Teachers should be aware of the importance of providing immediate, corrective, and positive feedback. Students' performance on regular progress checks, given throughout instruction, determines their level of mastery in terms of both their knowledge of the cognitive processes and self-regulation strategies and their application or performance on math problem-solving tests. Students graph their progress to visually display their performance, an activity that is very reinforcing for students as they can actually see their improvement over time. Careful analysis of performance during practice sessions and in mastery checks provides each student with honest feedback. Appropriate use of processes and strategies is reinforced continuously until students become proficient problem solvers. They need to know the specific behaviors for which they are praised so they can repeat these behaviors. Praise and reinforcement should be honest. Students should be taught how to give reinforcement to others and receive reinforcement from others. They need to have plenty of opportunities to practice giving and receiving reinforcement. The ultimate goal is to teach students how to monitor, evaluate, and reinforce themselves as problem solvers.

## Distributed Practice

Distributed practice is vital if students are to maintain what they have learned (Swanson, 1999). To become good math problem solvers, students learn to use the processes and strategies that successful problem solvers use. As a result, their math-problem-solving skills and performance levels improve. To achieve high performance levels, students must have many and varied opportunities to practice initially as they learn the math-problem-solving routine, and then, to maintain high performance, they must continue to practice intermittently over time. Practice can be individual or students can work in teams or small groups. Problems ranging from textbook to real-life problems should be included. Novel problems like the "school clothes" problem may take several problem-solving sessions. Following practice sessions, discussion about strategies, error monitoring, and alternative solutions is essential.

## Mastery Learning

A pretest is given before starting instruction to determine baseline performance levels of individual students. Then, throughout instruction, periodic mastery checks are given to monitor student progress over time and to determine the effectiveness of the program. If some students are not making sufficient progress, teachers must make modifications for these students to ensure success. Following instruction, periodic maintenance checks are provided. If students begin to slide in performance to the extent that they do not meet the required criteria on maintenance checks, booster sessions must be provided to return performance levels to mastery. These booster sessions are brief lessons consisting of review, practice, and a mastery check to refresh what students have previously learned and mastered.

## SOLVE IT!: A VALIDATED MATH PROBLEM-SOLVING PROGRAM

Solve It! (Montague, 2003) is a program specifically designed to teach students the cognitive processes and self-regulation strategies for math problem solving. It was designed to improve the problem solving of middle and secondary school students who have adequate reading and computational skills but still have difficulty solving math problems. Solve It! teaches students how good problem solvers solve math problems. The processes and strategies were identified through an extensive review of literature and a process–task analysis of problem solving. A math-problem-solving routine was developed and tested in a series of studies with middle and secondary students with learning disabilities (Montague, 1992; Montague et al., 1993; Montague & Bos, 1986). These studies demonstrated the effectiveness of the program with individual students and groups of between 8 and 12 students. Following instruction, the students with learning disabilities were compared with average-achieving peers and performed as well. Students appeared to maintain strategy use and improved performance for several weeks following instruction. Performance did decline over time for some students, but brief booster sessions consisting of review and practice helped them return to mastery level. The research-based program was designed to be easily embedded in a standard mathematics curriculum. Poor math problem solvers experience success at the outset and rapidly improve in problem-solving performance. In the studies, students developed a more positive attitude toward problem solving, a greater interest in mathematics and problem solving, independence as learners, and confidence in their ability to solve math problems.

To facilitate instruction, Solve It! provides sequenced and scripted lessons to ensure that the content is covered, and research-based instructional procedures are used. These scripts are meant to be adapted by teachers, if desired, to reflect teaching style and students' needs. The program explicitly teaches students how to apply the cognitive processes and self-regulation strategies in the context of math problem solving. Prior to implementation, students are given pretests to determine baseline performance level. Additionally, an informal assessment tool, the Math Problem Solving Assessment—Short Form (MPSA-SF), is included to analyze students' knowledge and use of problem-solving processes and strategies (Montague, 1996).

Initial assessment and ongoing monitoring of students' math problem solving enables teachers to measure individual students' performance before, during, and following instruction and ascertain each student's knowledge and use of processes and strategies. Assessment procedures like the MPSA-SF are designed to be student-centered, process-oriented, and directly relevant to the instructional program. Results give teachers an understanding of a student's knowledge base, skill level, learning style, information-processing strengths and weaknesses, strategic activity, attitude, and motivation for learning in a particular domain, like mathematics. They enable teachers to make judgments and informed decisions about both individual and group instructional needs.

## A SAMPLE SOLVE IT! LESSON

Solve It! lessons have instructional goals and behavioral objectives that are reflected in the content of the lesson. Each lesson lists the materials needed including instructional charts, practice problems, activities, and cue cards. Explicit instructional cues help the teacher pace the lesson by indicating which procedures to use and when to use them. The lesson script is divided into several steps. During Lessons 1–5, students learn the problem-solving routine (see Figure 5.1) and practice applying it. Practice sessions ensure that students' performance improves to a predetermined level (e.g., at least 70% correct on math-problem-solving mastery checks). Reinforcement and review are emphasized to help students maintain strategy use and improved performance over time. The criteria for moving to Lesson 6 are that all students in the group meet the mastery criterion (100%) for recitation of the cognitive processes from memory, that all students understand and are able to use the say, ask, check strategies, and that all students are able to work through practice problems with relative comfort and confidence. Students who do not meet criteria repeat Lessons 3–5. Practice sessions and progress checks are alternated until students meet the criteria for mastery. The following vignette illustrates how Solve It! is implemented in a general education math class.

### Mr. Sosa's Remedial Math Class

Mr. Sosa has 18 students in his seventh-grade remedial math class. Six students have identified learning disabilities and receive resource room support. All of the students have difficulty solving mathematical word problems. Mr. Sosa has been using Solve It! with these students. During Lessons 1–3, students were introduced to the processes and strategies, and they observed Mr. Sosa as he solved math problems. By Lesson 4, all students reached 100% of the criteria in recitation of the cognitive processes from memory. They also were comfortable with the say, ask, check procedures and were less reliant on the wall charts and their study booklets. Mr. Sosa had modeled problem solving for the students several times during the previous lessons. On occasion, individual students "guided" him through the process.

Mr. Sosa is beginning Lesson 4. He plans to model a solution one more time before students solve problems on their own.

He places a transparency of the math problem on the projector.

"Watch me say everything I am thinking and doing as I solve this problem.

There are eight boxes of Krispy Kreme donuts on the shelf. Each box holds 15 donuts. Artie comes in to pick up 45 donuts for his class party. How many donuts are left?

First, I am going to read the problem for understanding.

"SAY: Read the problem. OK, I will do that. [He reads the problem.] If I don't understand it, I will read it again. Hmm, I think I understand it, but let me just read it again to make sure. [He reads the problem again.]

"ASK: Have I read and understood the problem? Yes, definitely.

"CHECK: For understanding as I solve the problem. OK, I understand it.

"Next, I am going to paraphrase by putting the problem into my own words.

"SAY: Put the problem into my own words. This kid picks up 45 donuts. There are 8 boxes. Each box has 15 donuts. How many are left? Underline the important information. I will underline <u>8 boxes</u> and <u>Each box holds 15</u> and <u>pick up 45</u>.

"ASK: Have I underlined the important information? Let's see, yes I did. What is the question? The question is 'How many donuts are left?' What am I looking for? I am looking for the number of donuts left.

"CHECK: That the information goes with the question. I have the number of boxes, the number of donuts in each box, and the number that the kid took. I need to find how many are left.

"Then I will visualize by making a drawing or a diagram.

"SAY: Make a drawing or a diagram. Hmm, I will draw 8 boxes and write 15 in each box. Then, to the right I will write 'take away 45.'

"ASK: Does the picture fit the problem? Yes, I believe it does tell the story.

"CHECK: The picture against the problem information. Let me make sure I wrote the correct numbers: 8 boxes, 15, and 45. Yes, I did.

"Now I will hypothesize by making a plan to solve the problem.

"SAY: Decide how many steps and operations are needed. Let me see. First I need to get the total number of donuts in the boxes. Then I need to subtract the 45 that the kid took. OK, $8 \times 15$, and then subtract 45. OK. So, multiply and then subtract. Now I will write the operation symbols: $\times$, $-$.

"ASK: If I multiply $8 \times 15$, I will get the total number of donuts, and then I will subtract 45 from the total number and get the number of donuts left. How many steps are needed? 2 steps.

"CHECK: That the plan makes sense. If not, ask for help. It makes sense. Next I need to estimate by predicting the answer.

"SAY: Round the numbers, do the problem in my head, and write the estimate. Round 8 to 10 and then multiply by 15. That's easy: 150. Round 45 to 50 and subtract from 150, which is 100. There should be about 100 donuts left. Write 100.

"ASK: Did I round up and down? I rounded only up, but that's OK. Did I write the estimate? Yes.

"CHECK: That I used all the important information. Two steps. OK. Now I compute by doing the arithmetic.

"SAY: Do the operations in the right order. Okay, first multiply: 8 × 15. OK [does the arithmetic thinking aloud], 120. Then subtract: 120 – 45 [does the arithmetic thinking aloud]. That equals 75, my answer.

"ASK: How does my answer compare with my estimate? Hmm, not bad. I rounded up so my estimate would be more. Does my answer make sense? Yes, 75 donuts left. Are the decimals or money signs in the right places? None needed.

"CHECK: That all the operations were done in the right order. ×, –. Yes, they were. OK, now I really get to check to see if the answer is correct.

"SAY: Check the computation. Let's see. I will reverse the order to multiply and then check the subtraction by adding [demonstrates checking the computation].

"ASK: Have I checked every step? Yes. Have I used the right numbers [returns to the problem and checks the numbers again]. Yes, I have used the right numbers. Have I checked the computation? Yes, it's right. Is my answer right? Yes, the answer is right.

"CHECK: Now I will check myself again. I did everything correctly. The answer is right. I do not need to go back to the problem, and I do not need help."

Following the demonstration, students solve a problem on their own. They are told to use the processes and strategies and to think out loud just as the teacher did. They are also told to use their cue cards or refer to the Master Class Charts if they forget what to say and do. Mr. Sosa then selects a student to model the solution. He provides cues and prompts as needed to assist the student.

Like Mr. Sosa, teachers often ask how and when explicit strategy instruction should be provided for students with LD and also who should provide it. Research indicates that optimally, strategy instruction should be provided by expert remedial teachers who understand the characteristics of students with LD (Montague et al., 2000). Ideally, it should be provided to small groups of students (around 8 to 10), who have been assessed to determine that they will benefit from instruction. Grouping by need is important because some students may already be good problem solvers and may not need strategy instruction. Instruction is intense and time limited, so teachers may wish to remove students from the classroom for strategy instruction. Collaboration between general and special education teachers is essential if students are going to maintain and generalize what they have learned. Distributed practice and ongoing reinforcement are essential for long-term success.

These recommendations present several concerns surrounding the feasibility and practicality of providing cognitive strategy instruction. For instance, assessing students individually may not be possible with large groups of students. Individualizing instruction may be difficult, given the large numbers of students enrolled in most middle school teachers' math classes. Class size can range from 25 to 40 students, and teachers usually teach at least five classes. Enlisting the aid of the resource teacher to assist with instruction may be necessary. Identifying the students who need instruction and then grouping for instruction based on the various levels in the class can be a challenge for a math teacher. General education math teachers often feel unprepared to teach students who are in special programs. They may not feel confident that students can learn how to think differently and become good problem solvers. Finding time to talk with the resource teacher for students in special education can be difficult. Also, teachers often do not coordinate resource room instruction with the general education math curriculum. Communication between teachers sometimes can be difficult. Teachers may need to develop the knowledge and skills to implement cognitive strategy instruction successfully. Because the program is intense and highly interactive, teachers may need professional development to learn the instructional procedures that are the foundation of cognitive strategy instruction. Teachers may not be familiar with the research that supports cognitive strategy instruction and its components as well as the instructional procedures and, therefore, may not be convinced of its effectiveness.

## CONCLUSION

Solve It! is a research-based program that makes the cognitive processes and self-regulation strategies needed for mathematical problem solving easy to teach. Students are provided with the processes and strategies that make math problem solving easy to learn. With Solve It!, students learn how to self-regulate and become successful and efficient problem solvers. The ability to regulate one's performance is essential to success. As students become more successful, they gain a better attitude toward problem solving and develop the confidence to persevere. Moving from textbook problems to real-life math situations creates a challenge for students, and they begin to understand why they need to be good problem solvers. Research-based programs like Solve It! provide problem solving instruction that gives students the cognitive and self-regulation resources to solve authentic, complex mathematical problems they encounter in everyday life. Teachers who are knowledgeable about the research in cognitive strategy instruction will be able to justify the instructional time spent in their classes on programs like Solve It! They will also be able to explain how the program complements and builds on the mathematics curriculum.

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